1. Find the most general antiderivative of the function:

(a) $f(x) = x - 3$,

(b) $f(x) = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3$,

(c) $f(x) = \frac{1}{2}x^2 - 2x + 6$,

(d) $f(x) = 8x^9 - 3x^6 + 12x^3$,

(e) $g(t) = \frac{1 + t + t^2}{\sqrt{t}}$.

$f(x)$ $\quad \int f(x)$

x^n $\quad \frac{x^{n+1}}{n+1}$

a) $\int (x - 3)\, dx = \frac{x^{1+1}}{1+1} - 3x + C = \frac{x^2}{2} - 3x + C$

b) $\int \left(\frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3\right) dx = \frac{1}{2}x + \frac{3}{4}\frac{x^3}{3} - \frac{4}{5}\frac{x^4}{4} + C$

$= \frac{1}{2}x + \frac{x^3}{4} - \frac{x^4}{5} + C$

e) $g(t) = \frac{1 + t + t^2}{\sqrt{t}} = \frac{1}{\sqrt{t}} + \frac{t}{\sqrt{t}} + \frac{t^2}{\sqrt{t}}$ $\rightarrow t^{1/2}$

$= \frac{1}{t^{1/2}} + \frac{t}{t^{1/2}} + \frac{t^2}{t^{1/2}}$

$\frac{x^m}{x^n} = x^{m-n}$

$= t^{-1/2} + t^{1-1/2} + t^{2-1/2}$

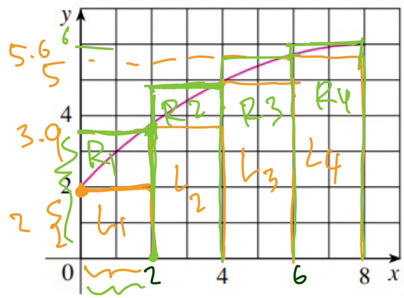
$= t^{-1/2} + t^{1/2} + t^{3/2}$

$\int \left(t^{-1/2} + t^{1/2} + t^{3/2}\right) dt =$

$\frac{t^{-1/2+1}}{-1/2+1} + \frac{t^{1/2+1}}{1/2+1} + \frac{t^{3/2+1}}{3/2+1} + C$

$= \frac{t^{1/2}}{1/2} + \frac{t^{3/2}}{3/2} + \frac{t^{5/2}}{5/2} + C$

2. By reading values from the given graph of f, use four rectangles to find a lower estimate and an upper estimate for the area under the given graph of f from $x = 0$ to $x = 8$.



lower estimate: $L_1 + L_2 + L_3 + L_4$

$4 + 7.8 + 10 + 11.2 = 33$

Upper estimate: $R_1 + R_2 + R_3 + R_4$

$7.8 + 10 + 11.2 + 12 = 41$

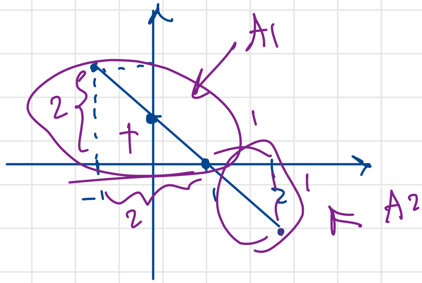
3. Evaluate the integral by interpreting it in terms of areas.

(a) $\int_{-1}^{2} \underbrace{(1-x)}_{f(x)}dx$,

(b) $\int_{0}^{9} \left(\frac{1}{3}x - 2\right) dx$.

a)

x	f(x)
0	1
1	0



$A_1 = \frac{2\times 2}{2} = 2$

$A_2 = \frac{1\times 1}{2} = \frac{1}{2}$

$A_1 - A_2 = 2 - \frac{1}{2} = 1.5$

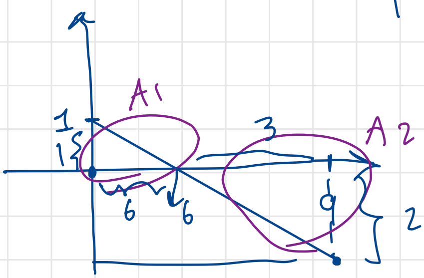
b) $\int_0^9 \underbrace{\left(\frac{1}{3}x - 2\right)}_{f(x)}dx$

x	f(x)
0	-2
9	1

$\frac{1}{3}x - 2 = 0$

$\frac{1}{3}x = 2$

$x = 6$



$A_1 = \frac{6\times 1}{2} = 3$

$A_2 = \frac{3\times 2}{2} = 3$

$A_1 - A_2 = 3 - 3 = 0$

4. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

(a) $y = \int_0^{x^4} \cos^2 \theta d\theta,$

(b) $g(x) = \int_3^{x} e^{t^2-t} dt,$

(c) $h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1} dz,$

(d) $y = \int_{1-3x}^{1} \frac{u^3}{1+u^2} du.$

$$f(x) = \int_a^x F(x)dx \Rightarrow f'(x) = F(x)$$

$$f(x) = \int_a^{g(x)} F(x)\,dx \Rightarrow f'(x) = F(g(x)) \cdot g'(x)$$

a) $\left(\int_0^{x^4} \cos^2\theta \, d\theta\right)' = \cos^2(x^4) \cdot 4x^3$

d) $\int_{1-3x}^{1} \frac{u^3}{1+u^2} du = -\int_1^{1-3x} \frac{u^3}{1+u^2} du$

$$= -\frac{(1-3x)^3}{1+(1-3x)^2} \cdot (-3)$$

$$= \frac{3(1-3x)^3}{1+(1-3x)^2}$$

5. Find the general indefinite integral:

(a) $\int \frac{x^3 - 2\sqrt{x}}{x} dx$

(b) $\int \left(x^4 - \frac{1}{2}x^3 + \frac{1}{4}x - 2\right) dx$

(c) $\int \left(\sqrt{x^3} + \sqrt[3]{x^2}\right) dx$

(d) $\int e^x + x^2 - \frac{2}{x} dx$

$$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$

a) $\int \frac{x^3 - 2\sqrt{x}}{x} dx = \int \left(\frac{x^3}{x} - \frac{2\sqrt{x}}{x} \right) dx$ ($\sqrt{x} \to x^{1/2}$)

$= \int (x^2 - 2x^{-1/2}) dx$

$= \frac{x^3}{3} - 2\frac{x^{1/2}}{1/2} + C$

$e^x \xrightarrow{\int} e^x$, $e^x \xrightarrow{} e^x$

$\ln(x) \xrightarrow{d/dx} 1/x$, $1/x \xrightarrow{\int} \ln(x)$

c) $\int (\sqrt{x^3} + \sqrt[3]{x^2}) dx = \int (x^{3/2} + x^{2/3}) dx$

$= \frac{x^{5/2}}{5/2} + \frac{x^{5/3}}{5/3} + C$

d) $\int e^x + x^2 - \underbrace{\frac{2}{x}}_{-2\left(\frac{1}{x}\right)} dx = e^x + \frac{x^3}{3} - 2\ln(x) + C$

6. If $\int_1^3 f(x)dx = 4$ and $\int_3^6 f(x)dx = 3$ and $\int_1^6 g(x)dx = 1$, what is $\int_1^6 2f(x) + 3g(x)dx$?

$a \leq c \leq b$

1) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

2) $\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$

3) $\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

$$\int_1^6 (2f(x) + 3g(x))\,dx$$

$$= \int_1^6 2f(x)\,dx + \int_1^6 3g(x)\,dx$$

$$= 2\int_1^6 f(x)\,dx + 3\underbrace{\int_1^6 g(x)\,dx}_{1}$$

$$= 2\left(\underbrace{\int_1^3 f(x)\,dx}_{4} + \underbrace{\int_3^6 f(x)\,dx}_{3}\right) + 3$$

$$= 2(4+3) + 3 = 2(7) + 3 = 14 + 3 = 17$$

7. Evaluate the indefinite integral:

(a) $\int e^x \sqrt{1+e^x}\,dx$,

(b) $\int x^2 e^{x^3}\,dx$,

(c) $\int \frac{(\ln x)^2}{x}\,dx$,

(d) $\int \frac{x^3}{1+x^4}\,dx$.

a) $\int_0^1 e^x \sqrt{1+e^x}\,dx$

$1+e^x = u$

$e^x dx = du$

$\int \sqrt{u}\,du = \int_2^{1+e} u^{1/2}\,du = \frac{u^{3/2}}{3/2} + C$

$= \frac{2}{3}(1+e^x)^{3/2} + C$

$x=0 \to u = 1+e^x = 1+e^0 = 1+1 = 2$

$x=1 \to u = 1+e$

$(e^u)' = u'e^u$

$e^{x^3} = u \to du = 3x^2 e^{x^3}\,dx$

b) $\int x^2 e^{x^3}\,dx$

$x^3 = u \to 3x^2 dx = du$

$\to dx = \frac{du}{3x^2}$

$= \int \cancel{x^2}\, e^u \frac{du}{3\cancel{x^2}} = \frac{1}{3}\int e^u du$

$= \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3} + C$

d) $\int_1^3 \frac{x^3\,dx}{1+x^4}$ $\frac{1}{3}du$

$1+x^4 = u$

$4x^3 dx = du$

$dx = \frac{du}{4x^3}$

$x=3 \to$

$u = 1+x^4$

$= 1+3^4 = 82$

$= \int \frac{\cancel{x^3}}{u} \frac{du}{4\cancel{x^3}} = \frac{1}{4}\int_2^{82} \frac{1}{u}\,du$

$x=1 \to$

$u = 1+1^4 = 2$

$= \frac{1}{4}\ln(u)\Big|_2^{82} = \frac{1}{4}(\ln(82) - \ln(2))$

$$\int_a^b \underbrace{f(x)}_{u}\,\underbrace{g(x)\,dx}_{du} = \int_{u(a)}^{u(b)} u\,du$$

$$= \int u\,du = V$$

$$\int_1^2 \frac{(\ln x)^2}{x}\,dx \qquad (\ln x)^2 \to u^2, \quad \frac{dx}{x} \to du$$

$\ln(x) = u \to \frac{1}{x}dx = du$

$x = 1 \to \ln(1)$

$x = 2 \to \ln(2)$

$$= \int_{u(1)}^{u(2)} u^2\,du = \frac{u^3}{3} + C = \frac{\ln(x)^3}{3}\Bigg|_1^2$$

$$= \frac{\ln(2)^3}{3} - \frac{\ln(1)^3}{3}$$

$$= \int_{\ln(1)}^{\ln(2)} u^2\,du = \frac{u^3}{3}\Bigg|_{\ln(1)}^{\ln(2)} = \frac{\ln(2)^3}{3} - \frac{\ln(1)^3}{3}$$

$$\int (x+3)(x-1)^5\,dx$$

$f(x) = x-1$
$g(x) = x^5$ $\Big\}$ $g(f(x))$

$$\int (u+4)(u)^5\,du$$

$u = x-1$
$du = dx$

$x+3 = (x-1)+4 = u+4$

$$= \int u^6 + 4u^5\,du$$

$$= \frac{u^7}{7} + 4\frac{u^6}{6} + C$$

$x=0 \rightarrow u = 0-1 = -1$
$x=2 \rightarrow u = 2-1 = 1$

$$\int_0^2 (x+3)(x-1)^5\,dx = \int_{u(0)}^{u(2)} u^6 + 4u^5\,du = \int_{-1}^{1} u^6 + 4u^5\,du$$

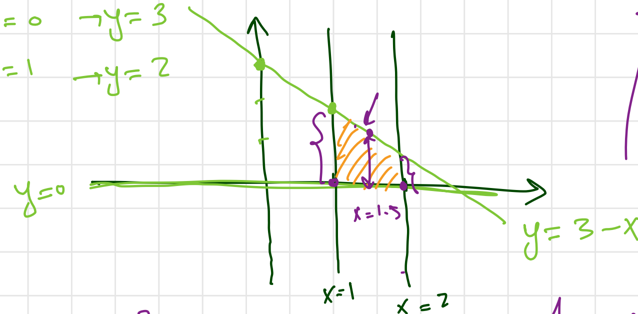
$$= \frac{u^7}{7} + 4\frac{u^6}{6}\Big|_{-1}^{1}$$

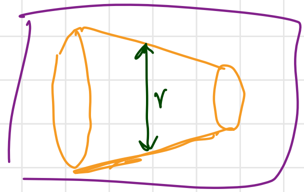
$$\frac{(x-1)^7}{7} + 4\frac{(x-1)^6}{6}\Big|_0^2$$

$y = 3 - x$, $y = 0$, $x = 1$, $x = 2$ about the x-axis

$x = 0 \rightarrow y = 3$

$x = 1 \rightarrow y = 2$





radius = $3 - x$

$A = \pi r^2 = \pi (3-x)^2$

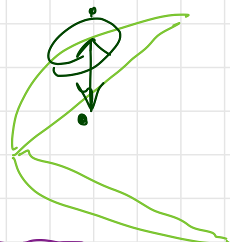
$$V = \int_1^2 A\, dx = \int_1^2 \pi (3-x)^2\, dx$$

$$= \int_1^2 \pi (9 - 6x + x^2)\, dx$$

$$= \pi \int_1^2 (9 - 6x + x^2)\, dx$$

$$= \pi \left(\underbrace{9x - 6\frac{x^2}{2} + \frac{x^3}{3}}_{f(x)} \right) \Bigg|_1^2$$

$$= \pi \left(f(2) - f(1) \right)$$



$y^2 = x$, $x = 2y$ about the y-axis

$g(y)$: $y^2 = x$ — 0, 2; $f(y)$: $x = 2y$ — 0, 1

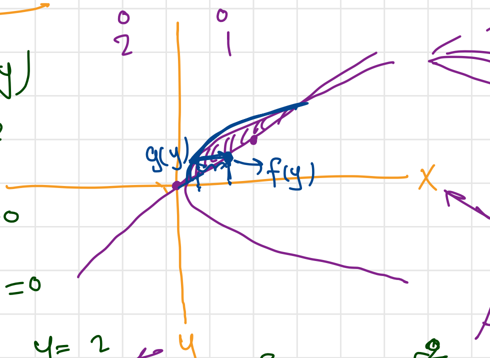
$f(y) = g(y)$

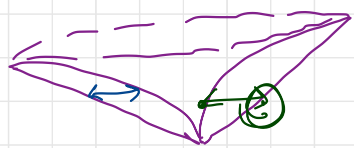
$2y = y^2$

$y^2 - 2y = 0$

$y(y-2) = 0$

$y = 0$ or $y = 2$





$f(y) - g(y)$

$= 2y - y^2 \rightarrow$ radius

$A = \pi r^2 = \pi (2y - y^2)^2$

$$V = \int A\, dy = \int_0^2 \pi (2y - y^2)^2\, dy$$