Projectile Motion:

Strategy

Again, resolving this two-dimensional motion into two independent one-dimensional motions allows us to solve for the desired quantities. The time a projectile is in the air is governed by its vertical motion alone. Thus, we solve for t first. While the ball is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, we recombine the vertical and horizontal results to obtain $\vec{v}$ at final time t, determined in the first part of the example.

Solution

(a) While the ball is in the air, it rises and then falls to a final position 10.0 m higher than its starting altitude. We can find the time for this by using Equation 4.22:

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2.$$

If we take the initial position y_0 to be zero, then the final position is $y = 10$ m. The initial vertical velocity is the vertical component of the initial velocity:

$$v_{0y} = v_0 \sin\theta_0 = (30.0\text{ m/s})\sin 45° = 21.2\text{ m/s}.$$

Substituting into Equation 4.22 for y gives us

$$10.0\text{ m} = (21.2\text{ m/s})t - (4.90\text{ m/s}^2)t^2.$$

Rearranging terms gives a quadratic equation in t:

$$(4.90\text{ m/s}^2)t^2 - (21.2\text{ m/s})t + 10.0\text{ m} = 0.$$

Use of the quadratic formula yields $t = 3.79$ s and $t = 0.54$ s. Since the ball is at a height of 10 m at two times during its trajectory—once on the way up and once on the way down—we take the longer solution for the time it takes the ball to reach the spectator:

$$t = 3.79\text{ s}.$$

The time for projectile motion is determined completely by the vertical motion. Thus, any projectile that has an initial vertical velocity of 21.2 m/s and lands 10.0 m above its starting altitude spends 3.79 s in the air.

(b) We can find the final horizontal and vertical velocities v_x and v_y with the use of the result from (a). Then, we can combine them to find the magnitude of the total velocity vector $\vec{\mathbf{v}}$ and the angle θ it makes with the horizontal. Since v_x is constant, we can solve for it at any horizontal location. We choose the starting point because we know both the initial velocity and the initial angle. Therefore,

$$v_x = v_0 \cos\theta_0 = (30\text{ m/s})\cos 45° = 21.2\text{ m/s}.$$

The final vertical velocity is given by Equation 4.21:

$$v_y = v_{0y} - gt.$$

Since v_{0y} was found in part (a) to be 21.2 m/s, we have

$$v_y = 21.2\text{ m/s} - 9.8\text{ m/s}^2(3.79\text{ s}) = -15.9\text{ m/s}.$$

The magnitude of the final velocity $\vec{\mathbf{v}}$ is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(21.2\text{ m/s})^2 + (-15.9\text{ m/s})^2} = 26.5\text{ m/s}.$$

The direction θ_v is found using the inverse tangent:

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-15.9}{21.2}\right) = 36.9° \text{ below the horizon.}$$

Significance

(a) As mentioned earlier, the time for projectile motion is determined completely by the vertical motion. Thus, any projectile that has an initial vertical velocity of 21.2 m/s and lands 10.0 m above its starting altitude spends 3.79 s in the air. (b) The negative angle means the velocity is $36.9°$ below the horizontal at the point of impact. This result is consistent with the fact that the ball is impacting at a point on the other side of the apex of the trajectory and therefore has a negative y component of the velocity. The magnitude of the velocity is less than the magnitude of the initial velocity we expect since it is impacting 10.0 m above the launch elevation.

Relative and Circular Motion:

Strategy

Given the speed of the jet, we can solve for the radius of the circle in the expression for the centripetal acceleration.

Solution

Set the centripetal acceleration equal to the acceleration of gravity: $9.8\ \text{m/s}^2 = v^2/r$.

Solving for the radius, we find

$$r = \frac{(134.1\ \text{m/s})^2}{9.8\ \text{m/s}^2} = 1835\ \text{m} = 1.835\ \text{km}.$$

Significance

To create a greater acceleration than *g* on the pilot, the jet would either have to decrease the radius of its circular trajectory or increase its speed on its existing trajectory or both.

Newton's Laws:

Strategy

In (a), we are considering the first part of Newton’s first law, dealing with a body at rest; in (b), we look at the second part of Newton’s first law for a body in motion.

Solution

a. When your car is parked, all forces on the car must be balanced; the vector sum is 0 N. Thus, the net force is zero, and Newton’s first law applies. The acceleration of the car is zero, and in this case, the velocity is also zero.
b. When your car is moving at constant velocity down the street, the net force must also be zero according to Newton’s first law. The car’s frictional force between the road and tires opposes the drag force on the car with the same magnitude, producing a net force of zero. The body continues in its state of constant velocity until the net force becomes nonzero. Realize that *a net force of zero means that an object is either at rest or moving with constant velocity, that is, it is not accelerating.* What do you suppose happens when the car accelerates? We explore this idea in the next section.

Significance

As this example shows, there are two kinds of equilibrium. In (a), the car is at rest; we say it is in *static equilibrium*. In (b), the forces on the car are balanced, but the car is moving; we say that it is in *dynamic equilibrium*. (We examine this idea in more detail in Static Equilibrium and Elasticity.) Again, it is possible for two (or more) forces to act on an object yet for the object to move. In addition, a net force of zero cannot produce acceleration.

Free Body Diagrams:

Figure 5.22 Since the acceleration is parallel to the slope and acting down the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular to it (axes shown to the left of the skier). $\vec{N}$ is perpendicular to the slope and $\vec{f}$ is parallel to the slope, but $\vec{w}$ has components along both axes, namely, w_y and w_x. Here, $\vec{w}$ has a squiggly line to show that it has been replaced by these components. The force $\vec{N}$ is equal in magnitude to w_y, so there is no acceleration perpendicular to the slope, but *f* is less than w_x, so there is a downslope acceleration (along the axis parallel to the slope).

Strategy

This is a two-dimensional problem, since not all forces on the skier (the system of interest) are parallel. The approach we have used in two-dimensional kinematics also works well here. Choose a convenient coordinate system and project the vectors onto its axes, creating two one-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Motions along mutually perpendicular axes are independent.) We use *x* and *y* for the parallel and perpendicular directions, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and the acceleration is downslope. Regarding the forces, friction is drawn in opposition to motion (friction always opposes forward motion) and is always parallel to the slope, w_x is drawn parallel to the slope and downslope (it causes the motion of the skier down the slope), and w_y is drawn as the component of weight perpendicular to the slope. Then, we can consider the separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the component of weight parallel to the slope is

$$w_x = w \sin 25° = mg \sin 25°,$$

and the magnitude of the component of the weight perpendicular to the slope is

$$w_y = w \cos 25° = mg \cos 25°.$$

a. Neglect friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the component of the skier's weight parallel to slope w_x and friction *f*. Using Newton's second law, with subscripts to denote quantities parallel to the slope,

$$a_x = \frac{F_{\text{net }x}}{m}$$

where $F_{\text{net }x} = w_x = mg \sin 25°$, assuming no friction for this part. Therefore,

$$a_x = \frac{F_{\text{net }x}}{m} = \frac{mg \sin 25°}{m} = g \sin 25°$$
$$\left(9.80 \text{ m/s}^2\right)(0.4226) = 4.14 \text{ m/s}^2$$

is the acceleration.

b. Include friction. We have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is

$$F_{\text{net }x} = w_x - f.$$

Substituting this into Newton's second law, $a_x = F_{\text{net }x}/m$, gives

$$a_x = \frac{F_{\text{net }x}}{m} = \frac{w_x - f}{m} = \frac{mg \sin 25° - f}{m}.$$

We substitute known values to obtain

$$a_x = \frac{(60.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(0.4226) - 45.0 \text{ N}}{60.0 \text{ kg}}.$$

This gives us

$$a_x = 3.39 \text{ m/s}^2,$$

which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

Significance

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. It is a general result that if friction on an incline is negligible, then the acceleration down the incline is $a = g \sin \theta$, regardless of mass. As discussed previously, all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).

Springs and Friction:

Figure 5.29 A spring exerts its force proportional to a displacement, whether it is compressed or stretched. (a) The spring is in a relaxed position and exerts no force on the block. (b) The spring is compressed by displacement Δx_1 of the object and exerts restoring force $-k\Delta x_1$. (c) The spring is stretched by displacement Δx_2 of the object and exerts restoring force $-k\Delta x_2$.

Work-Energy Theorem:

Strategy

The free-body diagram at the final position of the object is drawn in Figure 7.12. The gravitational work is the only work done over the displacement that is not zero. Since the weight points in the same direction as the net vertical displacement, the total work done by the gravitational force is positive. From the work-energy theorem, the starting height determines the speed of the car at the top of the loop,

$$-mg(y_2 - y_1) = \frac{1}{2}m{v_2}^2,$$

where the notation is shown in the accompanying figure. At the top of the loop, the normal force and gravity are both down and the acceleration is centripetal, so

$$a_{\text{top}} = \frac{F}{m} = \frac{N + mg}{m} = \frac{v_2^2}{R}.$$

The condition for maintaining contact with the track is that there must be some normal force, however slight; that is, $N > 0$. Substituting for v_2^2 and N, we can find the condition for y_1.

Solution

Implement the steps in the strategy to arrive at the desired result:

$$N = -mg + \frac{mv_2^2}{R} = \frac{-mgR + 2mg(y_1 - 2R)}{R} > 0 \quad \text{or} \quad y_1 > \frac{5R}{2}.$$

Significance

On the surface of the loop, the normal component of gravity and the normal contact force must provide the centripetal acceleration of the car going around the loop. The tangential component of gravity slows down or speeds up the car. A child would find out how high to start the car by trial and error, but now that you know the work-energy theorem, you can predict the minimum height (as well as other more useful results) from physical principles. By using the work-energy theorem, you did not have to solve a differential equation to determine the height.

Friction:

Strategy

We analyze the motions of the two blocks separately. The top block is subjected to a contact force exerted by the bottom block. The components of this force are the normal force N_1 and the frictional force $-0.400N_1$. Other forces on the top block are the tension $T\mathrm{i}$ in the string and the weight of the top block itself, 19.6 N. The bottom block is subjected to contact forces due to the top block and due to the floor. The first contact force has components $-N_1$ and $0.400N_1$, which are simply reaction forces to the contact forces that the bottom block exerts on the top block. The components of the contact force of the floor are N_2 and $0.400N_2$. Other forces on this block are $-P$, the tension $T\mathrm{i}$, and the weight –39.2 N.

Solution

Since the top block is moving horizontally to the right at constant velocity, its acceleration is zero in both the horizontal and the vertical directions. From Newton's second law,

$$\begin{aligned} \sum F_x &= m_1 a_x \qquad & \sum F_y &= m_1 a_y \\ T - 0.400N_1 &= 0 & N_1 - 19.6\,\mathrm{N} &= 0. \end{aligned}$$

Solving for the two unknowns, we obtain $N_1 = 19.6\,\mathrm{N}$ and $T = 0.40N_1 = 7.84\,\mathrm{N}$. The bottom block is also not accelerating, so the application of Newton's second law to this block gives

$$\begin{aligned} &\sum F_x = m_2 a_x & &\sum F_y = m_2 a_y \\ &T - P + 0.400\,N_1 + 0.400\,N_2 = 0 \qquad & &N_2 - 39.2\,\mathrm{N} - N_1 = 0. \end{aligned}$$

The values of N_1 and *T* were found with the first set of equations. When these values are substituted into the second set of equations, we can determine N_2 and *P*. They are

$$N_2 = 58.8\,\mathrm{N} \text{ and } P = 39.2\,\mathrm{N}.$$

Significance

Understanding what direction in which to draw the friction force is often troublesome. Notice that each friction force labeled in Figure 6.17 acts in the direction opposite the motion of its corresponding block.

Power

Strategy

At constant velocity, there is no change in kinetic energy, so the net work done to move the car is zero. Therefore the power supplied by the engine to move the car equals the power expended against gravity and air resistance. By assumption, 75% of the power is supplied against gravity, which equals $m\vec{g} \cdot \vec{v} = mgv \sin \theta$, where θ is the angle of the incline. A 15% grade means $\tan \theta = 0.15$. This reasoning allows us to solve for the power required.

Solution

Carrying out the suggested steps, we find

$$0.75\,P = mgv\sin(\tan^{-1} 0.15),$$

or

$$P = \frac{(1200\ \times\ 9.8\,\text{N})(90\,\text{m}/3.6\,\text{s})\sin(8.53°)}{0.75} = 58\,\text{kW},$$

or about 78 hp. (You should supply the steps used to convert units.)

Significance

This is a reasonable amount of power for the engine of a small to mid-size car to supply $(1\,\text{hp} = 0.746\,\text{kW})$. Note that this is only the power expended to move the car. Much of the engine's power goes elsewhere, for example, into waste heat. That's why cars need radiators. Any remaining power could be used for acceleration, or to operate the car's accessories.

Center of Mass

Center of Mass of the Earth-Moon System

Using data from text appendix, determine how far the center of mass of the Earth-moon system is from the center of Earth. Compare this distance to the radius of Earth, and comment on the result. Ignore the other objects in the solar system.

Strategy

We get the masses and separation distance of the Earth and moon, impose a coordinate system, and use Equation 9.29 with just $N = 2$ objects. We use a subscript "e" to refer to Earth, and subscript "m" to refer to the moon.

Solution

Define the origin of the coordinate system as the center of Earth. Then, with just two objects, Equation 9.29 becomes

$$R = \frac{m_e r_e + m_m r_m}{m_e + m_m}.$$

From Appendix D,

$$m_e = 5.97 \times 10^{24} \text{ kg}$$

$$m_m = 7.36 \times 10^{22} \text{ kg}$$

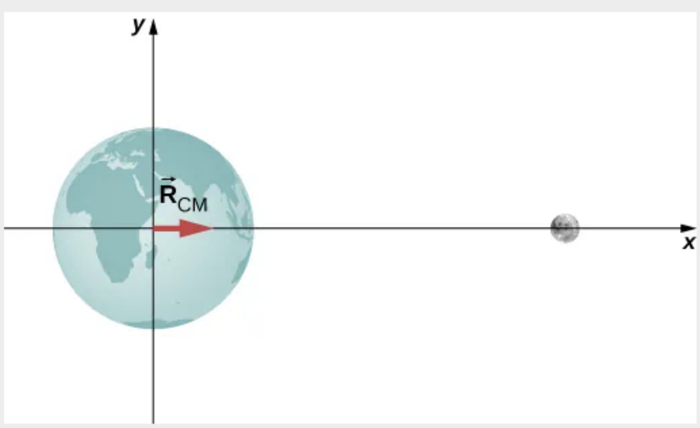
$$r_m = 3.82 \times 10^{8} \text{ m.}$$

We defined the center of Earth as the origin, so $r_e = 0$ m. Inserting these into the equation for *R* gives

$$\begin{aligned} R &= \frac{(5.97 \times 10^{24} \text{ kg})(0 \text{ m}) + (7.36 \times 10^{22} \text{ kg})(3.82 \times 10^{8} \text{ m})}{5.97 \times 10^{24} \text{ kg} + 7.36 \times 10^{22} \text{ kg}} \\ &= 4.64 \times 10^{6} \text{ m.} \end{aligned}$$

Significance

The radius of Earth is 6.37×10^6 m, so the center of mass of the Earth-moon system is (6.37 – 4.64) $\times 10^6 \text{ m} = 1.73 \times 10^6 \text{ m} = 1730 \text{ km}$ (roughly 1080 miles) *below* the surface of Earth. The location of the center of mass is shown (not to scale).



Conservation of Momentum

a. First, we posit conservation of momentum. For that, we need a closed system. The choice here is the system (hammer + Iron Man), from the time of collision to the moment just before Iron Man and the hammer hit the tree. Let:

   - M_{H} = mass of the hammer
   - M_{I} = mass of Iron Man
   - v_{H} = velocity of the hammer before hitting Iron Man
   - v = combined velocity of Iron Man + hammer after the collision

   Again, Iron Man's initial velocity was zero. Conservation of momentum here reads:

$$M_{\mathrm{H}} v_{\mathrm{H}} = (M_{\mathrm{H}} + M_{\mathrm{I}})\, v.$$

   We are asked to find the mass of the hammer, so we have

$$\begin{aligned} M_{\mathrm{H}} v_{\mathrm{H}} &= M_{\mathrm{H}} v + M_{\mathrm{I}} v \\ M_{\mathrm{H}}\,(v_{\mathrm{H}} - v) &= M_{\mathrm{I}} v \\ M_{\mathrm{H}} &= \frac{M_{\mathrm{I}} v}{v_{\mathrm{H}} - v} \\ &= \frac{(200\ \mathrm{kg})\left(\frac{2\ \mathrm{m}}{0.75\ \mathrm{s}}\right)}{10\ \frac{\mathrm{m}}{\mathrm{s}} - \left(\frac{2\ \mathrm{m}}{0.75\ \mathrm{s}}\right)} \\ &= 73\ \mathrm{kg}. \end{aligned}$$

   Considering the uncertainties in our estimates, this should be expressed with just one significant figure; thus, $M_{\mathrm{H}} = 7 \times 10^1$ kg.

b. The initial kinetic energy of the system, like the initial momentum, is all in the hammer:

$$\begin{aligned} K_{\mathrm{i}} &= \tfrac{1}{2} M_{\mathrm{H}} v_{\mathrm{H}}^2 \\ &= \tfrac{1}{2}(70\ \mathrm{kg})\,(10\ \mathrm{m/s})^2 \\ &= 3500\ \mathrm{J}. \end{aligned}$$

   After the collision,

$$\begin{aligned} K_{\mathrm{f}} &= \tfrac{1}{2}(M_{\mathrm{H}} + M_{\mathrm{I}})\, v^2 \\ &= \tfrac{1}{2}(70\ \mathrm{kg} + 200\ \mathrm{kg})\,(2.67\ \mathrm{m/s})^2 \\ &= 960\ \mathrm{J}. \end{aligned}$$

   Thus, there was a loss of $3500\ \mathrm{J} - 960\ \mathrm{J} = 2540\ \mathrm{J}$.

Inelastic Collisions

Define the system to be the two particles. This is a collision, so we should first identify what kind. Since we are told the two particles form a single particle after the collision, this means that the collision is perfectly inelastic. Thus, kinetic energy is not conserved, but momentum is. Thus, we use conservation of momentum to determine the final velocity of the system.

Solution

Treat the two particles as having identical masses *M*. Use the subscripts p, n, and d for proton, neutron, and deuteron, respectively. This is a one-dimensional problem, so we have

$$Mv_{\mathrm{p}} - Mv_{\mathrm{n}} = 2Mv_{\mathrm{d}}.$$

The masses divide out:

$$\begin{aligned} v_{\mathrm{p}} - v_{\mathrm{n}} &= 2v_{\mathrm{d}} \\ 7.0 \times 10^6 \text{ m/s} - 4.0 \times 10^6 \text{ m/s} &= 2v_{\mathrm{d}} \\ v_{\mathrm{d}} &= 1.5 \times 10^6 \text{ m/s}. \end{aligned}$$

The velocity is thus $\vec{\mathbf{v}}_{\mathrm{d}} = \left(1.5 \times 10^6 \text{ m/s}\right)\hat{\mathbf{i}}$.

Significance

This is essentially how particle colliders like the Large Hadron Collider work: They accelerate particles up to very high speeds (large momenta), but in opposite directions. This maximizes the creation of so-called “daughter particles.”

Strategy

The average angular acceleration can be found directly from its definition $\overline{\alpha} = \frac{\Delta\omega}{\Delta t}$ because the final angular velocity and time are given. We see that $\Delta\omega = \omega_{\text{final}} - \omega_{\text{initial}} = 250\ \text{rev/min}$ and Δt is 5.00 s. For part (b), we know the angular acceleration and the initial angular velocity. We can find the stopping time by using the definition of average angular acceleration and solving for Δt, yielding

$$\Delta t = \frac{\Delta\omega}{\alpha}.$$

Solution

a. Entering known information into the definition of angular acceleration, we get

$$\overline{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{250\ \text{rpm}}{5.00\ \text{s}}.$$

Because $\Delta\omega$ is in revolutions per minute (rpm) and we want the standard units of rad/s^2 for angular acceleration, we need to convert from rpm to rad/s:

$$\Delta\omega = 250\frac{\text{rev}}{\text{min}} \cdot \frac{2\pi\ \text{rad}}{\text{rev}} \cdot \frac{1\ \text{min}}{60\ \text{s}} = 26.2\frac{\text{rad}}{\text{s}}.$$

Entering this quantity into the expression for α, we get

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{26.2\ \text{rad/s}}{5.00\ \text{s}} = 5.24\ \text{rad/s}^2.$$

b. Here the angular velocity decreases from 26.2 rad/s (250 rpm) to zero, so that $\Delta\omega$ is –26.2 rad/s, and α is given to be –87.3 rad/s^2. Thus,

$$\Delta t = \frac{-26.2\ \text{rad/s}}{-87.3\ \text{rad/s}^2} = 0.300\ \text{s}.$$

Significance

Note that the angular acceleration as the mechanic spins the wheel is small and positive; it takes 5 s to produce an appreciable angular velocity. When she hits the brake, the angular acceleration is large and negative. The angular velocity quickly goes to zero.

Moment of Inertia

Strategy

a. We use the definition for moment of inertia for a system of particles and perform the summation to evaluate this quantity. The masses are all the same so we can pull that quantity in front of the summation symbol.
b. We do a similar calculation.
c. We insert the result from (a) into the expression for rotational kinetic energy.

Solution

a. $I = \sum_j m_j r_j^2 = (0.02\ \text{kg})(2\ \times\ (0.25\ \text{m})^2 + 2\ \times\ (0.15\ \text{m})^2 + 2\ \times\ (0.05\ \text{m})^2) = 0.0035\ \text{kg} \cdot \text{m}^2.$

b. $I = \sum_j m_j r_j^2 = (0.02\ \text{kg})(2\ \times\ (0.25\ \text{m})^2 + 2\ \times\ (0.15\ \text{m})^2) = 0.0034\ \text{kg} \cdot \text{m}^2.$

c. $K = \frac{1}{2} I\omega^2 = \frac{1}{2}(0.0035\ \text{kg} \cdot \text{m}^2)(5.0\ \times\ 2\pi\ \text{rad/s})^2 = 1.73\ \text{J}.$

Significance

We can see the individual contributions to the moment of inertia. The masses close to the axis of rotation have a very small contribution. When we removed them, it had a very small effect on the moment of inertia.

Torque

Strategy

We calculate each torque individually, using the cross product, and determine the sign of the torque. Then we sum the torques to find the net torque.

Solution

We start with $\vec{\mathbf{F}}_1$. If we look at Figure 10.35, we see that $\vec{\mathbf{F}}_1$ makes an angle of $90° + 60°$ with the radius vector $\vec{\mathbf{r}}$. Taking the cross product, we see that it is out of the page and so is positive. We also see this from calculating its magnitude:

$$\left|\vec{\boldsymbol{\tau}}_1\right| = rF_1 \sin 150° = 0.5\,\text{m}(20\,\text{N})(0.5) = 5.0\,\text{N}\cdot\text{m}.$$

Next we look at $\vec{\mathbf{F}}_2$. The angle between $\vec{\mathbf{F}}_2$ and $\vec{\mathbf{r}}$ is $90°$ and the cross product is into the page so the torque is negative. Its value is

$$\left|\vec{\boldsymbol{\tau}}_2\right| = -rF_2 \sin 90° = -0.5\,\text{m}(30\,\text{N}) = -15.0\,\text{N}\cdot\text{m}.$$

When we evaluate the torque due to $\vec{\mathbf{F}}_3$, we see that the angle it makes with $\vec{\mathbf{r}}$ is zero so $\vec{\mathbf{r}} \times \vec{\mathbf{F}}_3 = 0$. Therefore, $\vec{\mathbf{F}}_3$ does not produce any torque on the flywheel.

We evaluate the sum of the torques:

$$\tau_{\text{net}} = \sum_i |\boldsymbol{\tau}_i| = 5 - 15 = -10\,\text{N}\cdot\text{m}.$$

Significance

The axis of rotation is at the center of mass of the flywheel. Since the flywheel is on a fixed axis, it is not free to translate. If it were on a frictionless surface and not fixed in place, $\vec{\mathbf{F}}_3$ would cause the flywheel to translate, as well as $\vec{\mathbf{F}}_1$. Its motion would be a combination of translation and rotation.

Statics

$w_1 = m_1 g$ is the weight of mass m_1; $w_2 = m_2 g$ is the weight of mass m_2;

$w = mg$ is the weight of the entire meter stick; $w_3 = m_3 g$ is the weight of unknown mass m_3;

F_S is the normal reaction force at the support point *S*.

We choose a frame of reference where the direction of the *y*-axis is the direction of gravity, the direction of the *x*-axis is along the meter stick, and the axis of rotation (the *z*-axis) is perpendicular to the *x*-axis and passes through the support point *S*. In other words, we choose the pivot at the point where the meter stick touches the support. This is a natural choice for the pivot because this point does not move as the stick rotates. Now we are ready to set up the free-body diagram for the meter stick. We indicate the pivot and attach five vectors representing the five forces along the line representing the meter stick, locating the forces with respect to the pivot Figure 12.10. At this stage, we can identify the lever arms of the five forces given the information provided in the problem. For the three hanging masses, the problem is explicit about their locations along the stick, but the information about the location of the weight *w* is given implicitly. The key word here is "uniform." We know from our previous studies that the CM of a uniform stick is located at its midpoint, so this is where we attach the weight *w*, at the 50-cm mark.

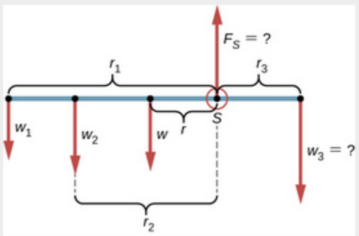


Figure 12.10 Free-body diagram for the meter stick. The pivot is chosen at the support point *S*.

Solution

With Figure 12.9 and Figure 12.10 for reference, we begin by finding the lever arms of the five forces acting on the stick:

$$\begin{aligned} r_1 &= 30.0\text{ cm} + 40.0\text{ cm} = 70.0\text{ cm} \\ r_2 &= 40.0\text{ cm} \\ r &= 50.0\text{ cm} - 30.0\text{ cm} = 20.0\text{ cm} \\ r_S &= 0.0\text{ cm (because } F_S \text{ is attached at the pivot)} \\ r_3 &= 30.0\text{ cm.} \end{aligned}$$

Now we can find the five torques with respect to the chosen pivot:

$$\begin{aligned} \tau_1 &= +r_1 w_1 \sin 90° = +r_1 m_1 g && \text{(counterclockwise rotation, positive sense)} \\ \tau_2 &= +r_2 w_2 \sin 90° = +r_2 m_2 g && \text{(counterclockwise rotation, positive sense)} \\ \tau &= +rw \sin 90° = +rmg && \text{(gravitational torque)} \\ \tau_S &= r_S F_S \sin \theta_S = 0 && \text{(because } r_S = 0\text{ cm)} \\ \tau_3 &= -r_3 w_3 \sin 90° = -r_3 m_3 g && \text{(clockwise rotation, negative sense)} \end{aligned}$$

The second equilibrium condition (equation for the torques) for the meter stick is

$$\tau_1 + \tau_2 + \tau + \tau_S + \tau_3 = 0.$$

When substituting torque values into this equation, we can omit the torques giving zero contributions. In this way the second equilibrium condition is

$$+r_1 m_1 g + r_2 m_2 g + rmg - r_3 m_3 g = 0. \tag{12.17}$$

Selecting the $+y$-direction to be parallel to $\vec{F}_S$, the first equilibrium condition for the stick is

$$-w_1 - w_2 - w + F_S - w_3 = 0.$$

Substituting the forces, the first equilibrium condition becomes

$$-m_1 g - m_2 g - mg + F_S - m_3 g = 0. \tag{12.18}$$

We solve these equations simultaneously for the unknown values m_3 and F_S. In Equation 12.17, we cancel the *g* factor and rearrange the terms to obtain

$$r_3 m_3 = r_1 m_1 + r_2 m_2 + rm.$$

To obtain m_3 we divide both sides by r_3, so we have

$$\begin{aligned} m_3 &= \frac{r_1}{r_3} m_1 + \frac{r_2}{r_3} m_2 + \frac{r}{r_3} m \\ &= \frac{70}{30}(50.0\text{ g}) + \frac{40}{30}(75.0\text{ g}) + \frac{20}{30}(150.0\text{ g}) = 316.0\tfrac{2}{3}\text{ g} \simeq 317\text{ g.} \end{aligned} \tag{12.19}$$

To find the normal reaction force, we rearrange the terms in Equation 12.18, converting grams to kilograms:

$$\begin{aligned} F_S &= (m_1 + m_2 + m + m_3)g \\ &= (50.0 + 75.0 + 150.0 + 316.7) \times 10^{-3}\text{kg} \times 9.8\,\tfrac{\text{m}}{\text{s}^2} = 5.8\text{ N.} \end{aligned} \tag{12.20}$$

Angular Momentum

Strategy

We resolve the acceleration into *x*- and *y*-components and use the kinematic equations to express the velocity as a function of acceleration and time. We insert these expressions into the linear momentum and then calculate the angular momentum using the cross-product. Since the position and momentum vectors are in the *xy*-plane, we expect the angular momentum vector to be along the *z*-axis. To find the torque, we take the time derivative of the angular momentum.

Solution

The meteor is entering Earth's atmosphere at an angle of $90.0°$ below the horizontal, so the components of the acceleration in the *x*- and *y*-directions are

$$a_x = 0, \quad a_y = -2.0\ \text{m/s}^2.$$

We write the velocities using the kinematic equations.

$$v_x = 0, \quad v_y = -2.0 \times 10^3\ \text{m/s} - (2.0\ \text{m/s}^2)t.$$

a. The angular momentum is

$$\begin{aligned}\vec{\mathbf{l}} &= \vec{\mathbf{r}} \times \vec{\mathbf{p}} = (25.0\ \text{km}\hat{\mathbf{i}} + 25.0\ \text{km}\hat{\mathbf{j}}) \times 15.0\ \text{kg}(0\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}) \\ &= 15.0\ \text{kg}[25.0\ \text{km}(v_y)\hat{\mathbf{k}}] \\ &= 15.0\ \text{kg}[2.50 \times 10^4\ \text{m}(-2.0 \times 10^3\ \text{m/s} - (2.0\ \text{m/s}^2)t)\hat{\mathbf{k}}].\end{aligned}$$

At $t = 0$, the angular momentum of the meteor about the origin is

$$\vec{\mathbf{l}}_0 = 15.0\ \text{kg}[2.50 \times 10^4\ \text{m}(-2.0 \times 10^3\ \text{m/s})\hat{\mathbf{k}}] = 7.50 \times 10^8\ \text{kg}\cdot\text{m}^2/\text{s}(-\hat{\mathbf{k}}).$$

This is the instant that the observer sees the meteor.

b. To find the torque, we take the time derivative of the angular momentum. Taking the time derivative of $\vec{\mathbf{l}}$ as a function of time, which is the second equation immediately above, we have

$$\frac{d\vec{\mathbf{l}}}{dt} = -15.0\ \text{kg}(2.50 \times 10^4\ \text{m})(2.0\ \text{m/s}^2)\hat{\mathbf{k}}.$$

Then, since $\frac{d\vec{\mathbf{l}}}{dt} = \sum \vec{\boldsymbol{\tau}}$, we have

$$\sum \vec{\boldsymbol{\tau}} = -7.5 \times 10^5\text{N}\cdot\text{m}\hat{\mathbf{k}}.$$

The units of torque are given as newton-meters, not to be confused with joules. As a check, we note that the lever arm is the *x*-component of the vector $\vec{\mathbf{r}}$ in Figure 11.10 since it is perpendicular to the force acting on the meteor, which is along its path. By Newton's second law, this force is

$$\vec{\mathbf{F}} = ma(-\hat{\mathbf{j}}) = 15.0\ \text{kg}(2.0\ \text{m/s}^2)(-\hat{\mathbf{j}}) = 30.0\ \text{kg}\cdot\text{m/s}^2(-\hat{\mathbf{j}}).$$

The lever arm is

$$\vec{\mathbf{r}}_\perp = 2.5 \times 10^4\ \text{m}\ \hat{\mathbf{i}}.$$

Thus, the torque is

$$\begin{aligned}\sum \vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}_\perp \times \vec{\mathbf{F}} &= (2.5 \times 10^4\ \text{m}\ \hat{\mathbf{i}}) \times (-30.0\ \text{kg}\cdot\text{m/s}^2\hat{\mathbf{j}}), \\ &= 7.5 \times 10^5\ \text{N}\cdot\text{m}(-\hat{\mathbf{k}}).\end{aligned}$$

Significance

Since the meteor is accelerating downward toward Earth, its radius and velocity vector are changing. Therefore, since $\vec{\mathbf{l}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$, the angular momentum is changing as a function of time. The torque on the meteor about the origin, however, is constant, because the lever arm $\vec{\mathbf{r}}_\perp$ and the force on the meteor are constants. This example is important in that it illustrates that the angular momentum depends on the choice of origin about which it is calculated. The methods used in this example are also important in developing angular momentum for a system of particles and for a rigid body.

Gravitation

Answer: This question asks for a change in potential energy. We will use the symbol Δ to mean "the change in". So, we will use the equation to find ΔU, the change in gravitational potential energy:

$\Delta U = U_{final} - U_{initial}$

$$\Delta U = \left(-G\frac{(2400\ kg)(100000\ kg)}{100\ \mathrm{m}}\right) - \left(-G\ \frac{(2400\ kg)(100000\ kg)}{100\ m}\right)$$

Take out common factors

$$\Delta U = -\mathrm{G}(240000\ kg)(100000\ kg)\left(\frac{1}{1000\ m} - \frac{1}{100\ m}\right)$$

$$\Delta \mathrm{U} = -\mathrm{G}\ (240000\ kg)\ (100000\ kg)\left(-\frac{9}{1000\ m}\right)$$

$$\Delta U = -(6.673 \times 10^{-11}\ \mathrm{N} \cdot m^2/kg^2)(240000\ kg)(100000\ kg)\left(-\frac{9}{1000 m}\right)$$

$\Delta U = 0.0144\ \mathrm{N \cdot m}$

$\Delta U = 0.0144\ \mathrm{J}$

The change in gravitational potential energy as the asteroids move away from each other is 0.0144 Joules.

Harmonic Motion

Strategy

We first find the angular frequency. The phase shift is zero, $\phi = 0.00 \text{ rad}$, because the block is released from rest at $x = A = +0.02 \text{ m}$. Once the angular frequency is found, we can determine the maximum velocity and maximum acceleration.

Solution

The angular frequency can be found and used to find the maximum velocity and maximum acceleration:

$$\begin{aligned} \omega &= \frac{2\pi}{1.57 \text{ s}} = 4.00 \text{ s}^{-1}; \\ v_{\max} &= A\omega = 0.02\text{m} \left(4.00 \text{ s}^{-1}\right) = 0.08 \text{ m/s}; \\ a_{\max} &= A\omega^2 = 0.02 \text{ m}\left(4.00 \text{ s}^{-1}\right)^2 = 0.32 \text{ m/s}^2. \end{aligned}$$

All that is left is to fill in the equations of motion:

$$\begin{aligned} x(t) &= A\cos(\omega t + \phi) = (0.02 \text{ m})\cos\left(4.00 \text{ s}^{-1} t\right); \\ v(t) &= -v_{\max}\sin(\omega t + \phi) = (-0.08 \text{ m/s})\sin\left(4.00 \text{ s}^{-1} t\right); \\ a(t) &= -a_{\max}\cos(\omega t + \phi) = \left(-0.32 \text{ m/s}^2\right)\cos\left(4.00 \text{ s}^{-1} t\right). \end{aligned}$$

Significance

The position, velocity, and acceleration can be found for any time. It is important to remember that when using these equations, your calculator must be in radians mode.

Pendulums

Strategy

We are asked to find *g* given the period *T* and the length *L* of a pendulum. We can solve $T = 2\pi\sqrt{\frac{L}{g}}$ for *g*, assuming only that the angle of deflection is less than $15°$.

Solution

1. Square $T = 2\pi\sqrt{\frac{L}{g}}$ and solve for *g*:

$$g = 4\pi^2 \frac{L}{T^2}.$$

2. Substitute known values into the new equation:

$$g = 4\pi^2 \frac{0.75000\ \text{m}}{(1.7357\ \text{s})^2}.$$

3. Calculate to find *g*:

$$g = 9.8281\ \text{m/s}^2.$$

Significance

This method for determining *g* can be very accurate, which is why length and period are given to five digits in this example. For the precision of the approximation $\sin\theta \approx \theta$ to be better than the precision of the pendulum length and period, the maximum displacement angle should be kept below about $0.5°$.

Waves

Strategy

a. The speed of the wave can be derived by dividing the distance traveled by the time.
b. The period of the wave is the inverse of the frequency of the driving force.
c. The wavelength can be found from the speed and the period $v = \lambda/T$.

Solution

a. The first wave traveled 30.00 m in 6.00 s:

$$v = \frac{30.00 \text{ m}}{6.00 \text{ s}} = 5.00\frac{\text{m}}{\text{s}}.$$

b. The period is equal to the inverse of the frequency:

$$T = \frac{1}{f} = \frac{1}{2.00 \text{ s}^{-1}} = 0.50 \text{ s}.$$

c. The wavelength is equal to the velocity times the period:

$$\lambda = vT = 5.00\frac{\text{m}}{\text{s}}(0.50 \text{ s}) = 2.50 \text{ m}.$$

Significance

The frequency of the wave produced by an oscillating driving force is equal to the frequency of the driving force.

Bernoulli's

Strategy

We must use Bernoulli's equation to solve for the pressure, since depth is not constant.

Solution

Bernoulli's equation is

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

where subscripts 1 and 2 refer to the initial conditions at ground level and the final conditions inside the nozzle, respectively. We must first find the speeds v_1 and v_2. Since $Q = A_1 v_1$, we get

$$v_1 = \frac{Q}{A_1} = \frac{40.0 \times 10^{-3} \text{m}^3/\text{s}}{\pi(3.20 \times 10^{-2} \text{m})^2} = 12.4 \text{m/s}.$$

Similarly, we find

$$v_2 = 56.6 \text{ m/s}.$$

This rather large speed is helpful in reaching the fire. Now, taking h_1 to be zero, we solve Bernoulli's equation for p_2:

$$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho g h_2.$$

Substituting known values yields

$$\begin{aligned} p_2 &= 1.62 \times 10^6 \text{ N/m}^2 + \tfrac{1}{2}(1000 \text{ kg/m}^3)[(12.4 \text{ m/s})^2 - (56.6 \text{ m/s})^2] \\ &\quad - (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(10.0 \text{ m}) \\ &= -2.9 \text{ kPa} \approx 0 \text{ kPa (when compared to air pressure).} \end{aligned}$$

Significance

This value is a gauge pressure, since the initial pressure was given as a gauge pressure. Thus, the nozzle pressure equals atmospheric pressure because the water exits into the atmosphere without changes in its conditions.