Ohms Law:

V=IR

Find delta Vs

Kirchoffs Law:

I1-I2-I3 = 0

V-IR=0

RC Circuits:

Short at start, open at inf

Magnetic Force:

Strategy

From the free-body diagram in the figure, the tensions in the supporting leads go to zero when the gravitational and magnetic forces balance each other. Using the RHR-1, we find that the magnetic force points up. We can then determine the current *I* by equating the two forces.

Solution

Equate the two forces of weight and magnetic force on the wire:

$$mg = IlB.$$

Thus,

$$I = \frac{mg}{lB} = \frac{(0.010\ \text{kg})(9.8\ \text{m/s}^2)}{(0.50\ \text{m})(0.50\ \text{T})} = 0.39\ \text{A}.$$

Significance

This large magnetic field creates a significant force on a length of wire to counteract the weight of the wire.

Force on a Current Loop

Strategy

The dipole moment is defined by the current times the area of the loop. The area of the loop can be calculated from the area of the circle. The torque on the loop and potential energy are calculated from identifying the magnetic moment, magnetic field, and angle oriented in the field.

Solution

a. The magnetic moment μ is calculated by the current times the area of the loop or πr^2.

$$\mu = IA = (2.0 \times 10^{-3}\ \text{A})(\pi(0.02\ \text{m})^2) = 2.5 \times 10^{-6}\ \text{A} \cdot \text{m}^2$$

b. The torque and potential energy are calculated by identifying the magnetic moment, magnetic field, and the angle between these two vectors. The calculations of these quantities are:

$$\begin{aligned}
\tau &= \left|\vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}\right| = \mu B \sin\theta = \left(2.5 \times 10^{-6}\text{A} \cdot \text{m}^2\right)(0.50\text{T})\sin(30°) = 6.3 \times 10^{-7}\text{N} \cdot \text{m} \\
U &= -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}} = -\mu B\cos\theta = -\left(2.5 \times 10^{-6}\text{A} \cdot \text{m}^2\right)(0.50\text{T})\cos(30°) = -1.1 \times 10^{-6}\text{J}.
\end{aligned}$$

Significance

The concept of magnetic moment at the atomic level is discussed in the next chapter. The concept of aligning the magnetic moment with the magnetic field is the functionality of devices like magnetic motors, whereby switching the external magnetic field results in a constant spinning of the loop as it tries to align with the field to minimize its potential energy.

Biot-Savart Law

Strategy

We can determine the magnetic field at point *P* using the Biot-Savart law. Since the current segment is much smaller than the distance *x*, we can drop the integral from the expression. The integration is converted back into a summation, but only for small *dl*, which we now write as Δl. Another way to think about it is that each of the radius values is nearly the same, no matter where the current element is on the line segment, if Δl is small compared to *x*. The angle θ is calculated using a tangent function. Using the numbers given, we can calculate the magnetic field at *P*.

Solution

The angle between $\Delta\vec{\mathbf{l}}$ and $\hat{\mathbf{r}}$ is calculated from trigonometry, knowing the distances *l* and *x* from the problem:

$$\theta = \tan^{-1}\left(\frac{1\text{ m}}{0.01\text{ m}}\right) = 89.4°.$$

The magnetic field at point *P* is calculated by the Biot-Savart law:

$$B = \frac{\mu_0}{4\pi}\frac{I\Delta l\sin\theta}{r^2} = (1 \times 10^{-7}\text{T}\cdot\text{m/A})\left(\frac{2\text{ A}(0.01\text{ m})\sin(89.4°)}{(1\text{ m})^2}\right) = 2.0 \times 10^{-9}\text{T}.$$

From the right-hand rule and the Biot-Savart law, the field is directed into the page.

Significance

This approximation is only good if the length of the line segment is very small compared to the distance from the current element to the point. If not, the integral form of the Biot-Savart law must be used over the entire line segment to calculate the magnetic field.

Magnetic Fields of Current Loops

Strategy

The magnetic field at point *P* has been determined in Equation 12.15. Since the currents are flowing in opposite directions, the net magnetic field is the difference between the two fields generated by the coils. Using the given quantities in the problem, the net magnetic field is then calculated.

Solution

Solving for the net magnetic field using Equation 12.15 and the given quantities in the problem yields

$$B = \frac{\mu_0 I R_1^2}{2\left(y_1^2+R_1^2\right)^{3/2}} - \frac{\mu_0 I R_2^2}{2\left(y_2^2+R_2^2\right)^{3/2}}$$

$$B = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(0.010\ \text{A})(0.5\ \text{m})^2}{2((0.25\ \text{m})^2+(0.5\ \text{m})^2)^{3/2}} - \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(0.010\ \text{A})(1.0\ \text{m})^2}{2((0.75\ \text{m})^2+(1.0\ \text{m})^2)^{3/2}}$$

$$B = 5.77 \times 10^{-9}\,\text{T to the right.}$$

Significance

Helmholtz coils typically have loops with equal radii with current flowing in the same direction to have a strong uniform field at the midpoint between the loops. A similar application of the magnetic field distribution created by Helmholtz coils is found in a magnetic bottle that can temporarily trap charged particles. See Magnetic Forces and Fields for a discussion on this.

Ampere’s Law

Solution

For any circular path of radius *r* that is centered on the wire,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \oint B dl = B \oint dl = B(2\pi r).$$

From Ampère's law, this equals the total current passing through any surface bounded by the path of integration.

Consider first a circular path that is inside the wire ($r \leq a$) such as that shown in part (a) of Figure 12.16. We need the current *I* passing through the area enclosed by the path. It's equal to the current density *J* times the area enclosed. Since the current is uniform, the current density inside the path equals the current density in the whole wire, which is $I_0/\pi a^2$. Therefore the current *I* passing through the area enclosed by the path is

$$I = \frac{\pi r^2}{\pi a^2} I_0 = \frac{r^2}{a^2} I_0.$$

We can consider this ratio because the current density *J* is constant over the area of the wire. Therefore, the current density of a part of the wire is equal to the current density in the whole area. Using Ampère's law, we obtain

$$B(2\pi r) = \mu_0 \left(\frac{r^2}{a^2} \right) I_0,$$

and the magnetic field inside the wire is

$$B = \frac{\mu_0 I_0}{2\pi} \frac{r}{a^2} \; (r \leq a).$$

Outside the wire, the situation is identical to that of the infinite thin wire of the previous example; that is,

$$B = \frac{\mu_0 I_0}{2\pi r} \; (r \geq a).$$

The variation of *B* with *r* is shown in Figure 12.17.

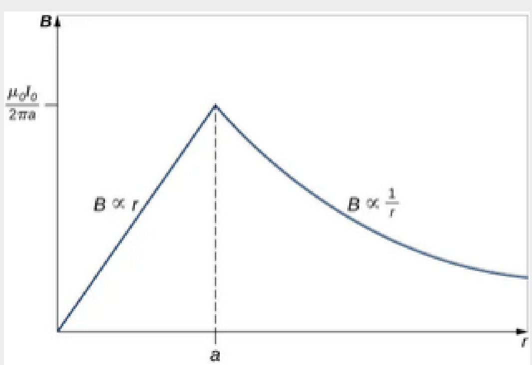

Figure 12.17 Variation of the magnetic field produced by a current I_0 in a long, straight wire of radius a.

Significance

The results show that as the radial distance increases inside the thick wire, the magnetic field increases from zero to a familiar value of the magnetic field of a thin wire. Outside the wire, the field drops off regardless of whether it was a thick or thin wire.

This result is similar to how Gauss's law for electrical charges behaves inside a uniform charge distribution, except that Gauss's law for electrical charges has a uniform volume distribution of charge, whereas Ampère's law here has a uniform area of current distribution. Also, the drop-off outside the thick wire is similar to how an electric field drops off outside of a linear charge distribution, since the two cases have the same geometry and neither case depends on the configuration of charges or currents once the loop is outside the distribution.