Friction:

Strategy

We analyze the motions of the two blocks separately. The top block is subjected to a contact force exerted by the bottom block. The components of this force are the normal force N_1 and the frictional force $-0.400N_1$. Other forces on the top block are the tension $T\mathrm{i}$ in the string and the weight of the top block itself, 19.6 N. The bottom block is subjected to contact forces due to the top block and due to the floor. The first contact force has components $-N_1$ and $0.400N_1$, which are simply reaction forces to the contact forces that the bottom block exerts on the top block. The components of the contact force of the floor are N_2 and $0.400N_2$. Other forces on this block are $-P$, the tension $T\mathrm{i}$, and the weight –39.2 N.

Solution

Since the top block is moving horizontally to the right at constant velocity, its acceleration is zero in both the horizontal and the vertical directions. From Newton's second law,

$$\begin{aligned} \sum F_x &= m_1 a_x & \qquad \sum F_y &= m_1 a_y \\ T - 0.400 N_1 &= 0 & N_1 - 19.6\,\mathrm{N} &= 0. \end{aligned}$$

Solving for the two unknowns, we obtain $N_1 = 19.6\,\mathrm{N}$ and $T = 0.40N_1 = 7.84\,\mathrm{N}$. The bottom block is also not accelerating, so the application of Newton's second law to this block gives

$$\begin{aligned} &\sum F_x = m_2 a_x & &\sum F_y = m_2 a_y \\ &T - P + 0.400\,N_1 + 0.400\,N_2 = 0 & &N_2 - 39.2\,\mathrm{N} - N_1 = 0. \end{aligned}$$

The values of N_1 and *T* were found with the first set of equations. When these values are substituted into the second set of equations, we can determine N_2 and *P*. They are

$$N_2 = 58.8\,\mathrm{N} \ \text{ and } \ P = 39.2\,\mathrm{N}.$$

Significance

Understanding what direction in which to draw the friction force is often troublesome. Notice that each friction force labeled in Figure 6.17 acts in the direction opposite the motion of its corresponding block.

Power

Strategy

At constant velocity, there is no change in kinetic energy, so the net work done to move the car is zero. Therefore the power supplied by the engine to move the car equals the power expended against gravity and air resistance. By assumption, 75% of the power is supplied against gravity, which equals $m\vec{\mathbf{g}} \cdot \vec{\mathbf{v}} = mgv \sin\theta$, where θ is the angle of the incline. A 15% grade means $\tan\theta = 0.15$. This reasoning allows us to solve for the power required.

Solution

Carrying out the suggested steps, we find

$$0.75\,P = mgv\,\sin(\tan^{-1} 0.15),$$

or

$$P = \frac{(1200\,\times\,9.8\text{ N})(90\text{ m}/3.6\text{ s})\sin(8.53°)}{0.75} = 58\text{ kW},$$

or about 78 hp. (You should supply the steps used to convert units.)

Significance

This is a reasonable amount of power for the engine of a small to mid-size car to supply $(1\text{ hp} = 0.746\text{ kW})$. Note that this is only the power expended to move the car. Much of the engine's power goes elsewhere, for example, into waste heat. That's why cars need radiators. Any remaining power could be used for acceleration, or to operate the car's accessories.

Center of Mass

Center of Mass of the Earth-Moon System

Using data from text appendix, determine how far the center of mass of the Earth-moon system is from the center of Earth. Compare this distance to the radius of Earth, and comment on the result. Ignore the other objects in the solar system.

Strategy

We get the masses and separation distance of the Earth and moon, impose a coordinate system, and use Equation 9.29 with just $N = 2$ objects. We use a subscript "e" to refer to Earth, and subscript "m" to refer to the moon.

Solution

Define the origin of the coordinate system as the center of Earth. Then, with just two objects, Equation 9.29 becomes

$$R = \frac{m_e r_e + m_m r_m}{m_e + m_m}.$$

From Appendix D,

$$m_e = 5.97 \times 10^{24}\ \text{kg}$$

$$m_m = 7.36 \times 10^{22}\ \text{kg}$$

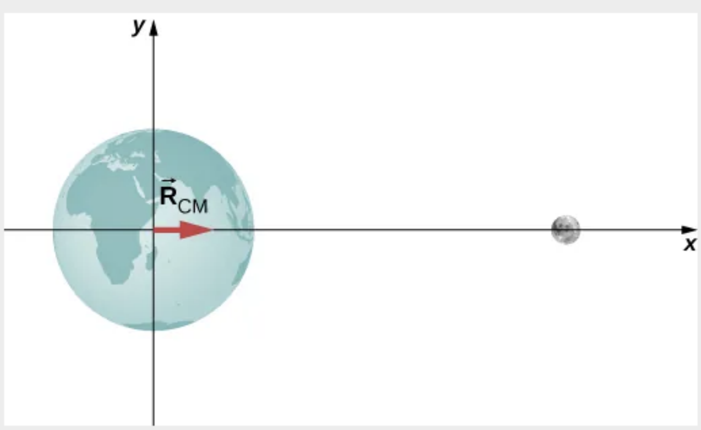
$$r_m = 3.82 \times 10^{8}\ \text{m}.$$

We defined the center of Earth as the origin, so $r_e = 0\ \text{m}$. Inserting these into the equation for *R* gives

$$\begin{aligned} R &= \frac{(5.97 \times 10^{24}\ \text{kg})(0\ \text{m}) + (7.36 \times 10^{22}\ \text{kg})(3.82 \times 10^{8}\ \text{m})}{5.97 \times 10^{24}\ \text{kg} + 7.36 \times 10^{22}\ \text{kg}} \\ &= 4.64 \times 10^{6}\ \text{m}. \end{aligned}$$

Significance

The radius of Earth is $6.37 \times 10^6\ \text{m}$, so the center of mass of the Earth-moon system is (6.37 – 4.64) $\times 10^6\ \text{m} = 1.73 \times 10^6\ \text{m} = 1730\ \text{km}$ (roughly 1080 miles) *below* the surface of Earth. The location of the center of mass is shown (not to scale).



Conservation of Momentum

a. First, we posit conservation of momentum. For that, we need a closed system. The choice here is the system (hammer + Iron Man), from the time of collision to the moment just before Iron Man and the hammer hit the tree. Let:

- M_{H} = mass of the hammer
- M_{I} = mass of Iron Man
- v_{H} = velocity of the hammer before hitting Iron Man
- v = combined velocity of Iron Man + hammer after the collision

Again, Iron Man's initial velocity was zero. Conservation of momentum here reads:

$$M_{\mathrm{H}} v_{\mathrm{H}} = (M_{\mathrm{H}} + M_{\mathrm{I}})\, v.$$

We are asked to find the mass of the hammer, so we have

$$\begin{aligned} M_{\mathrm{H}} v_{\mathrm{H}} &= M_{\mathrm{H}} v + M_{\mathrm{I}} v \\ M_{\mathrm{H}} (v_{\mathrm{H}} - v) &= M_{\mathrm{I}} v \\ M_{\mathrm{H}} &= \frac{M_{\mathrm{I}} v}{v_{\mathrm{H}} - v} \\ &= \frac{(200\ \mathrm{kg})\left(\frac{2\ \mathrm{m}}{0.75\ \mathrm{s}}\right)}{10\ \frac{\mathrm{m}}{\mathrm{s}} - \left(\frac{2\ \mathrm{m}}{0.75\ \mathrm{s}}\right)} \\ &= 73\ \mathrm{kg}. \end{aligned}$$

Considering the uncertainties in our estimates, this should be expressed with just one significant figure; thus, $M_{\mathrm{H}} = 7 \times 10^{1}\ \mathrm{kg}$.

b. The initial kinetic energy of the system, like the initial momentum, is all in the hammer:

$$\begin{aligned} K_{\mathrm{i}} &= \tfrac{1}{2} M_{\mathrm{H}} v_{\mathrm{H}}^{2} \\ &= \tfrac{1}{2} (70\ \mathrm{kg})\,(10\ \mathrm{m/s})^{2} \\ &= 3500\ \mathrm{J}. \end{aligned}$$

After the collision,

$$\begin{aligned} K_{\mathrm{f}} &= \tfrac{1}{2} (M_{\mathrm{H}} + M_{\mathrm{I}})\, v^{2} \\ &= \tfrac{1}{2} (70\ \mathrm{kg} + 200\ \mathrm{kg})\,(2.67\ \mathrm{m/s})^{2} \\ &= 960\ \mathrm{J}. \end{aligned}$$

Thus, there was a loss of $3500\ \mathrm{J} - 960\ \mathrm{J} = 2540\ \mathrm{J}$.

Inelastic Collisions

Define the system to be the two particles. This is a collision, so we should first identify what kind. Since we are told the two particles form a single particle after the collision, this means that the collision is perfectly inelastic. Thus, kinetic energy is not conserved, but momentum is. Thus, we use conservation of momentum to determine the final velocity of the system.

Solution

Treat the two particles as having identical masses *M*. Use the subscripts p, n, and d for proton, neutron, and deuteron, respectively. This is a one-dimensional problem, so we have

$$Mv_{\text{p}} - Mv_{\text{n}} = 2Mv_{\text{d}}.$$

The masses divide out:

$$\begin{aligned} v_{\text{p}} - v_{\text{n}} &= 2v_{\text{d}} \\ 7.0 \times 10^6 \text{ m/s} - 4.0 \times 10^6 \text{ m/s} &= 2v_{\text{d}} \\ v_{\text{d}} &= 1.5 \times 10^6 \text{ m/s.} \end{aligned}$$

The velocity is thus $\vec{\mathbf{v}}_{\text{d}} = \left(1.5 \times 10^6 \text{ m/s}\right) \hat{\mathbf{i}}$.

Significance

This is essentially how particle colliders like the Large Hadron Collider work: They accelerate particles up to very high speeds (large momenta), but in opposite directions. This maximizes the creation of so-called "daughter particles."

Strategy

The average angular acceleration can be found directly from its definition $\overline{\alpha} = \frac{\Delta\omega}{\Delta t}$ because the final angular velocity and time are given. We see that $\Delta\omega = \omega_{\text{final}} - \omega_{\text{initial}} = 250\ \text{rev/min}$ and Δt is 5.00 s. For part (b), we know the angular acceleration and the initial angular velocity. We can find the stopping time by using the definition of average angular acceleration and solving for Δt, yielding

$$\Delta t = \frac{\Delta\omega}{\alpha}.$$

Solution

a. Entering known information into the definition of angular acceleration, we get

$$\overline{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{250\ \text{rpm}}{5.00\ \text{s}}.$$

Because $\Delta\omega$ is in revolutions per minute (rpm) and we want the standard units of rad/s^2 for angular acceleration, we need to convert from rpm to rad/s:

$$\Delta\omega = 250\frac{\text{rev}}{\text{min}} \cdot \frac{2\pi\ \text{rad}}{\text{rev}} \cdot \frac{1\ \text{min}}{60\ \text{s}} = 26.2\frac{\text{rad}}{\text{s}}.$$

Entering this quantity into the expression for α, we get

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{26.2\ \text{rad/s}}{5.00\ \text{s}} = 5.24\ \text{rad/s}^2.$$

b. Here the angular velocity decreases from 26.2 rad/s (250 rpm) to zero, so that $\Delta\omega$ is –26.2 rad/s, and α is given to be $-87.3\ \text{rad/s}^2$. Thus,

$$\Delta t = \frac{-26.2\ \text{rad/s}}{-87.3\ \text{rad/s}^2} = 0.300\ \text{s}.$$

Significance

Note that the angular acceleration as the mechanic spins the wheel is small and positive; it takes 5 s to produce an appreciable angular velocity. When she hits the brake, the angular acceleration is large and negative. The angular velocity quickly goes to zero.

Moment of Inertia

Strategy

a. We use the definition for moment of inertia for a system of particles and perform the summation to evaluate this quantity. The masses are all the same so we can pull that quantity in front of the summation symbol.
b. We do a similar calculation.
c. We insert the result from (a) into the expression for rotational kinetic energy.

Solution

a. $I = \sum_j m_j r_j^2 = (0.02\ \text{kg})(2 \times (0.25\ \text{m})^2 + 2 \times (0.15\ \text{m})^2 + 2 \times (0.05\ \text{m})^2) = 0.0035\ \text{kg} \cdot \text{m}^2.$
b. $I = \sum_j m_j r_j^2 = (0.02\ \text{kg})(2 \times (0.25\ \text{m})^2 + 2 \times (0.15\ \text{m})^2) = 0.0034\ \text{kg} \cdot \text{m}^2.$
c. $K = \frac{1}{2} I\omega^2 = \frac{1}{2}(0.0035\ \text{kg} \cdot \text{m}^2)(5.0 \times 2\pi\ \text{rad/s})^2 = 1.73\ \text{J}.$

Significance

We can see the individual contributions to the moment of inertia. The masses close to the axis of rotation have a very small contribution. When we removed them, it had a very small effect on the moment of inertia.

Torque

Strategy

We calculate each torque individually, using the cross product, and determine the sign of the torque. Then we sum the torques to find the net torque.

Solution

We start with $\vec{\mathbf{F}}_1$. If we look at Figure 10.35, we see that $\vec{\mathbf{F}}_1$ makes an angle of $90° + 60°$ with the radius vector $\vec{\mathbf{r}}$. Taking the cross product, we see that it is out of the page and so is positive. We also see this from calculating its magnitude:

$$\left|\vec{\boldsymbol{\tau}}_1\right| = rF_1 \sin 150° = 0.5\text{ m}(20\text{ N})(0.5) = 5.0\text{ N} \cdot \text{m}.$$

Next we look at $\vec{\mathbf{F}}_2$. The angle between $\vec{\mathbf{F}}_2$ and $\vec{\mathbf{r}}$ is $90°$ and the cross product is into the page so the torque is negative. Its value is

$$\left|\vec{\boldsymbol{\tau}}_2\right| = -rF_2 \sin 90° = -0.5\text{ m}(30\text{ N}) = -15.0\text{ N} \cdot \text{m}.$$

When we evaluate the torque due to $\vec{\mathbf{F}}_3$, we see that the angle it makes with $\vec{\mathbf{r}}$ is zero so $\vec{\mathbf{r}} \times \vec{\mathbf{F}}_3 = 0.$ Therefore, $\vec{\mathbf{F}}_3$ does not produce any torque on the flywheel.

We evaluate the sum of the torques:

$$\tau_{\text{net}} = \sum_i |\boldsymbol{\tau}_i| = 5 - 15 = -10\text{ N} \cdot \text{m}.$$

Significance

The axis of rotation is at the center of mass of the flywheel. Since the flywheel is on a fixed axis, it is not free to translate. If it were on a frictionless surface and not fixed in place, $\vec{\mathbf{F}}_3$ would cause the flywheel to translate, as well as $\vec{\mathbf{F}}_1$. Its motion would be a combination of translation and rotation.