1. Use the graph of $f(x)$ in the following figure to determine each of the following values:

   (a) $\lim_{x\to -4^-} f(x)$; $\lim_{x\to -4^+} f(x)$; $\lim_{x\to -4} f(x)$; $f(-4)$

   0 0 0 0

   (b) $\lim_{x\to -2^-} f(x)$; $\lim_{x\to -2^+} f(x)$; $\lim_{x\to -2} f(x)$; $f(-2)$

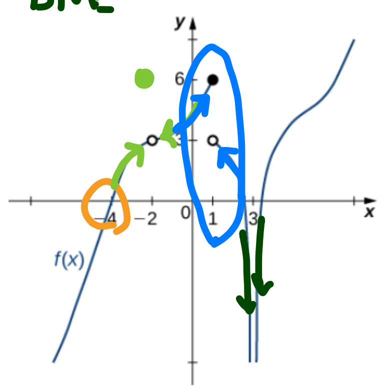
   3 3 3 DNE

   (c) $\lim_{x\to 1^-} f(x)$; $\lim_{x\to 1^+} f(x)$; $\lim_{x\to 1} f(x)$; $f(1)$

   6 3 DNE 6

   (d) $\lim_{x\to 3^-} f(x)$; $\lim_{x\to 3^+} f(x)$; $\lim_{x\to 3} f(x)$; $f(3)$

   $-\infty$ $-\infty$ $-\infty$ DNE



$\dfrac{\text{number}}{0}$ $\pm\infty$

2. Evaluate each of the following limits:

   (a) $\lim_{x\to 5^+} \dfrac{x+1}{x-5} = \dfrac{5+1}{5-5} = \dfrac{6}{0}$ $\qquad \dfrac{5.1+1}{5.1-5} = \dfrac{6.1}{0.1} = +\infty$

   (b) $\lim_{x\to 5^-} \dfrac{x+1}{x-5} = \dfrac{4.9+1}{4.9-5} = \dfrac{5.9}{-0.1} = -\infty$ $\qquad \dfrac{0}{0}$

   (c) $\lim_{x\to 2^-} \dfrac{x^2-2x}{x^2-4x+4}$

$(x+a)(x+b) = x^2+(a+b)x+ab$

$$= \lim_{x\to 2^-} \frac{x\cancel{(x-2)}}{(x-2)^{\cancel{2}}} = \lim_{x\to 2^-} \frac{x}{x-2} = -\infty$$

$(x-2)(x-2)$ $\qquad 1.9 \qquad \dfrac{1.9}{1.9-2} = \dfrac{1.9}{-0.1}$

3. Find the vertical asymptotes of the function $y = \dfrac{x^2+1}{3x-2x^2}$.

$x=a$ is a V.A if $\lim_{x\to a} f(x) = \pm\infty$

find the zeroes of the denom.

$3x-2x^2=0 \to x(3-2x)=0$

$x=0$ or $3-2x=0 \to 3=2x$

$\to x = \frac{3}{2}$

$$\lim_{x \to 0} \frac{x^2+1}{3x-2x^2} = \frac{0+1}{0} \longrightarrow \infty \qquad x=0 \text{ is a V.A}$$

$$\lim_{x \to \frac{3}{2}} \frac{x^2+1}{3x-2x^2} = \frac{\frac{9}{4}+1}{0} \longrightarrow \infty \qquad x=\frac{3}{2} \text{ is a V.A}$$

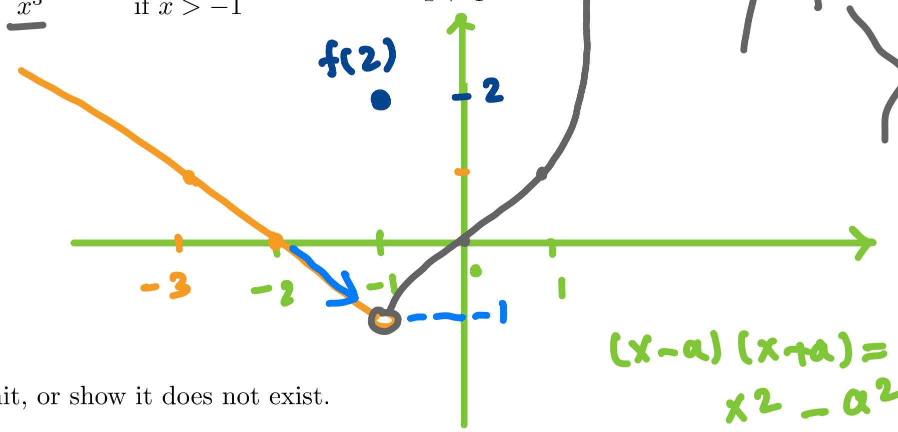
$$f(x) = \frac{x^2-1}{x+1} \qquad \lim_{x \to -1} \frac{x^2-1}{x+1} = \lim_{x \to -1} \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}}$$

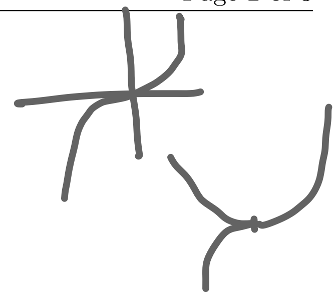
$$x = -1$$

$$= \lim_{x \to -1} x-1 = -2$$

x	y
-2	$-(-2)-2=0$
-3	$-(-3)-2=1$

4. Graph $f(x) = \begin{cases} -x-2 & \text{if } x < -1 \\ 2 & \text{if } x = -1 \\ x^3 & \text{if } x > -1 \end{cases}$ and evaluate $\lim\limits_{x \to -1^-} f(x)$. -1





$(x-a)(x+a) = x^2 - a^2$

5. Evaluate the limit, or show it does not exist.

(a) $\lim\limits_{x \to 5} \dfrac{x^2 - 6x + 5}{x - 5} = 4$

(b) $\lim\limits_{t \to 0} \dfrac{\sqrt{1+t} - \sqrt{1-t}}{t}$ $\times \dfrac{\sqrt{1+t} + \sqrt{1-t}}{\sqrt{1+t} + \sqrt{1-t}} = \lim\limits_{t \to 0} \dfrac{(\sqrt{1+t})^2 - (\sqrt{1-t})^2}{t(\sqrt{1+t} + \sqrt{1-t})}$

(c) $\lim\limits_{x \to -4} \dfrac{\sqrt{x^2+9} - 5}{x + 4} = \dfrac{-8}{10}$

(d) $\lim\limits_{x \to 16} \dfrac{4 - \sqrt{x}}{16x - x^2}$

$1+t-1+t$

$= \lim\limits_{t \to 0} \dfrac{1+t-(1-t)}{t(\sqrt{1+t} + \sqrt{1-t})}$

$= \lim\limits_{t \to 0} \dfrac{2\cancel{t}}{\cancel{t}(\sqrt{1+t} + \sqrt{1-t})}$

$= \lim\limits_{t \to 0} \dfrac{2}{\sqrt{1+t} + \sqrt{1-t}} = \dfrac{2}{1+1} = 1$

6. Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither? Sketch the graph of f.

$$f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ x & \text{if } -1 \leqslant x < 1 \\ 1/x & \text{if } x \geqslant 1 \end{cases}$$

$\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^-} x^2 = (-1)^2 = 1$

$\neq$

$\lim\limits_{x \to -1^+} f(x) = \lim\limits_{x \to -1^+} x = -1$

$\lim\limits_{x \to -1} f(x) = \text{DNE}$

$f(x)$ is discont. at $x = -1$

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} x = 1$

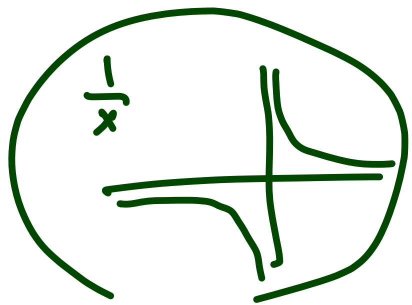
$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \dfrac{1}{x} = 1$

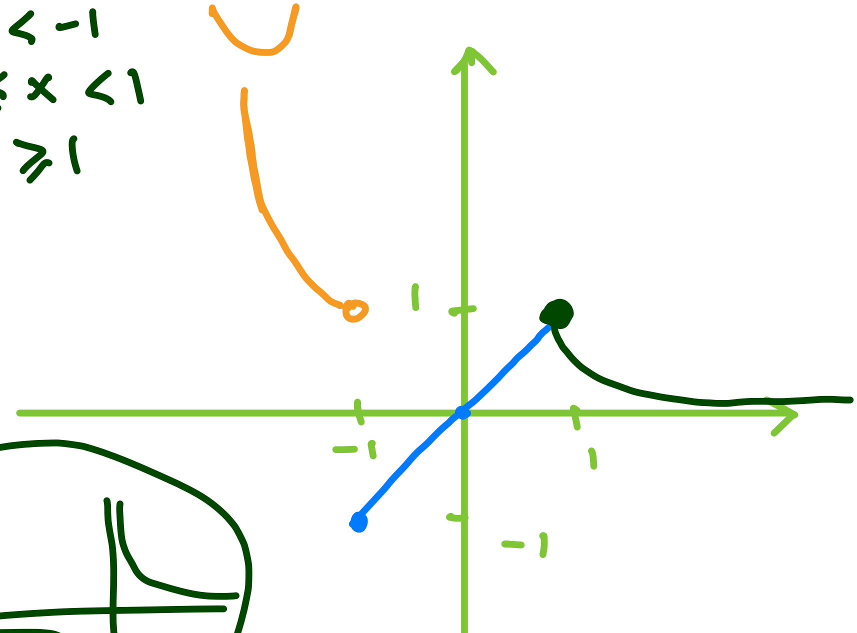
$\lim\limits_{x \to 1} f(x) = 1$

Cont.

$$f(x) = \begin{cases} x^2 & x < -1 \\ x & -1 \leq x < 1 \\ \frac{1}{x} & x \geq 1 \end{cases}$$

x	f(x)
-1	-1
0	0





5(b) $\frac{0}{0}$ $\lim\limits_{x \to 16} \dfrac{4 - \sqrt{x}}{16x - x^2}$

$$\frac{4 - \sqrt{16}^{\,4}}{\underbrace{16 \times 16 - 16^2}_{0}} = \frac{0}{0}$$

16^2

$$= \lim_{x \to 16} \frac{4 - \sqrt{x}}{x(16 - x)}$$

$4^2 \qquad (\sqrt{x})^2$

$$= \lim_{x \to 16} \frac{\cancel{4 - \sqrt{x}}}{x(\cancel{4 - \sqrt{x}})(4 + \sqrt{x})} = \lim_{x \to 16} \frac{1}{x(4 + \sqrt{x})}$$

$$= \frac{1}{16(4 + \sqrt{16})} = \boxed{\frac{1}{16(4+4)}} = \frac{1}{16 \times 8} = \boxed{\frac{1}{128}}$$

$\lim (f(x) + g(x)) = \lim f(x) + \lim g(x)$

$\rightarrow \lim f(x)\, g(x) = \lim f(x) \cdot \lim g(x)$

7. Suppose f and g are continuous functions such that $g(2) = 6$ and $\lim_{x\to 2} (3f(x) + f(x)g(x)) = 36$. Find $f(2)$.

$$\lim_{x\to 2} (3f(x) + f(x)g(x)) = \lim_{x\to 2} 3f(x) + \lim_{x\to 2} f(x)g(x)$$

$$= 3\lim_{x\to 2} f(x) + \lim_{x\to 2} f(x) \cdot \underbrace{\lim_{x\to 2} g(x)}_{6} = 36$$

$9\lim_{x\to 2} f(x) = 36$

$f(x) = 4$

8. Use the Intermediate Value Theorem to show that $y = x^4 + x - 3$ has a root on the interval $(1, 2)$.

$= 0$

$f(1) = 1^4 + 1 - 3 = -1$

$f(2) = 2^4 + 2 - 3 = 15$

$-1 \leq 0 \leq 15$ ✓

9. Find the limit or show that it does not exist:

(a) $\lim_{x\to\infty} \dfrac{3x-2}{2x+1} = \dfrac{3}{2}$

(b) $\lim_{x\to\infty} \dfrac{x^4 - 3x^2 + x}{x^3 - x + 2} = \infty$

(c) $\lim_{x\to-\infty} \dfrac{1-x^2}{x^3 - x + 1} = 0$

10. Find the horizontal and vertical asymptotes of the graphs of the following functions:

	V.A	H.A
(a) $y = \dfrac{5+4x}{x+3}$	$x = -3$	$y = 4$
(b) $y = \dfrac{2x^2+1}{3x^2+2x-1}$	$x = \frac{1}{3}$, $x = -1$	$y = \frac{2}{3}$
(c) $y = \dfrac{2e^x}{e^x - 5}$	$x = \ln(5)$	$y = 0$, $y = 2$

The End.