Confidence Intervals and Test Statistics

Sampling Conditions	Confidence Interval	Test Statistic
Is n is large, say over 30?		
Yes		
σ^2 is known $\Rightarrow$ $\bar{X}$ ~ N (μ, σ^2/n)	$\bar{X} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$	$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$
σ^2 is unknown $\Rightarrow$ $\bar{X}$ ~ N (μ, σ^2/n)	$\bar{X} \pm Z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$	$Z = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$
	$\bar{p} \pm Z_{\alpha/2}\left(\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right)$	$Z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$
No		
X ~ N (μ, σ^2) and σ^2 is known $\Rightarrow$ $\bar{X}$ ~ N (μ, σ^2/n)	$\bar{X} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$	$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$
X ~ N (μ, σ^2) and σ^2 is unknown $\Rightarrow$ $\bar{X}$ ~ t_{n-1} (μ, σ^2/n)	$\bar{X} \pm t_{n-1,\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$	$t_{n-1} = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$

Population Proportion

Parameter p

Statistic $\hat{p}$

Standard Error

$$\text{s.e.}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Confidence Interval

$$\hat{p} \pm z^* \text{s.e.}(\hat{p})$$

Conservative Confidence Interval

$$\hat{p} \pm \frac{z^*}{2\sqrt{n}}$$

Large-Sample *z*-Test

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Sample Size

$$n = \left(\frac{z^*}{2m}\right)^2$$

Two Population Proportions

Parameter $p_1 - p_2$

Statistic $\hat{p}_1 - \hat{p}_2$

Standard Error

$$\text{s.e.}(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Confidence Interval

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \text{s.e.}(\hat{p}_1 - \hat{p}_2)$$

Large-Sample *z*-Test

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$

Population Mean

Parameter μ

Statistic $\bar{x}$

Standard Error

$$\text{s.e.}(\bar{x}) = \frac{s}{\sqrt{n}}$$

Confidence Interval

$$\bar{x} \pm t^* \text{s.e.}(\bar{x}) \qquad \text{df} = n - 1$$

Paired Confidence Interval

$$\bar{d} \pm t^* \text{s.e.}(\bar{d}) \qquad \text{df} = n - 1$$

One-Sample *t*-Test

$$t = \frac{\bar{x} - \mu_0}{\text{s.e.}(\bar{x})} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \qquad \text{df} = n - 1$$

Paired *t*-Test

$$t = \frac{\bar{d} - 0}{\text{s.e.}(\bar{d})} = \frac{\bar{d}}{s_d/\sqrt{n}} \qquad \text{df} = n - 1$$